AC Power Flow and Mixed-Integer Nonlinear Optimization

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AC Power Flow & Integer Nonlinear Optimization

1. Applications of Integer Nonlinear Optimization

2. Software for Integer Nonlinear Optimization

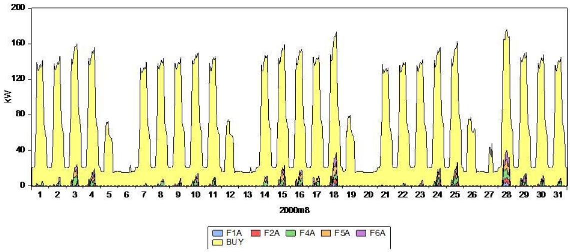
3. Branch-and-Refine for Nonconvex MINLPs

4. Conclusions

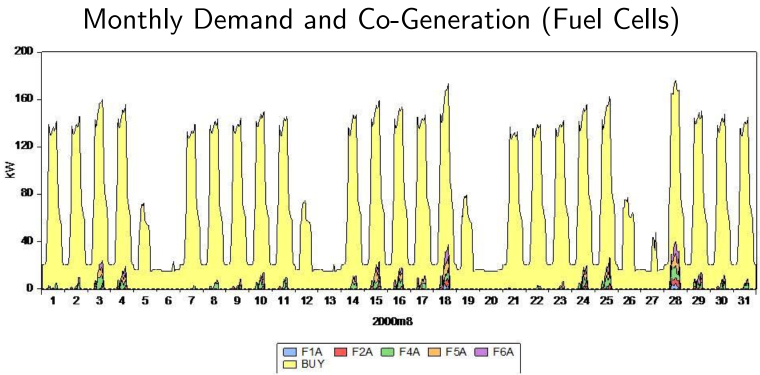
Co-Generation for Commercial Buildings

With Colorado School of Mines (Pruit, Newman & Braun) & NREL.

- Co-generation units: fuel-cell, solar panel, wind, storage unit.
- Which units should we buy to minimize energy and purchase cost? Binary variables model type of equipment & size (discrete).
- Ramping constraints for fuel-cell & storage unit ⇒ nonlinearities.
- Operate the units optimally; includes on/off constraints.



Co-Generation for Commercial Buildings



- Fuel-cells reduce demand spikes (penalty). $\Rightarrow$ Reduces uncertainty?!
- Only single scenario; need stochastic optimization.
  $\Rightarrow$ Storage may be more important without perfect foresight!

Tertiary Voltage Control: Nonlinearities

AC power flow equations (U voltage; θ angles)

$$P_k = \sum_{l:(k,l)\in\mathcal{A}} \left(b_{kl} U_k U_l \sin(\theta_k - \theta_l) + g_{kl} U_k^2 - g_{kl} U_k U_l \cos(\theta_k - \theta_l) \right)$$

- Nonconvex, nonlinear system of equations.
- Challenge for global optimization solvers: $\sin(x), \cos(x)$!

Define new variables $e_k = U_k \sin\theta_k$ and $f_k = V_k \cos\theta_k$
Use $\cos(\theta_k - \theta_l) = \cos\theta_k \cos\theta_l + \sin\theta_k \sin\theta_l$ to get:

$$P_k = \sum_{l:(k,l)\in\mathcal{A}} \left(b_{kl}(f_k e_l - e_k f_l) + g_{kl}(e_k^2 + f_k^2) - g_{kl}(e_k e_l + f_k f_l) \right)$$

- Polynomial system of equations $\Rightarrow$ global solution possible.
- 662-Bus AC optimal power flow solves in 2 seconds (KNITRO-IPM).

Tertiary Voltage Control: Discrete Decisions

Transformer Settings:

- Proportional to ratio of number of wire turns.
- Model with special-ordered set: $r \in \{R_1, \ldots, R_N\}$

$$r = \sum_{i=1}^{N} R_i y_i, \quad 1 = \sum_{i=1}^{N} y_i, \quad y_i \in \{0,1\}$$

- Branch on special-ordered set, not on individual y_i!
  If $R_k < r < R_{k+1}$, then create two child nodes:

$$\begin{array}{ccc} r \le R_k & \text{and} & r \ge R_{k+1} \\ \hline \displaystyle\sum_{i=k+1}^{N} y_i = 0 & & \displaystyle\sum_{i=1}^{k} y_i = 0 \end{array}$$

Tertiary Voltage Control: Discrete Decisions

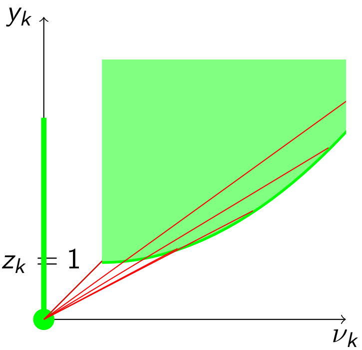
Capacitor Banks

- Decision whether stored reactive power is released or not.
- Single binary variable $z_k = 1$ if released, $z_k = 0$ if not.
- Kirchhoff's law becomes ...

$$Q_k + z_k \nu_k^2 Q_{0k} - \sum_{l:(k,l)\in\mathcal{A}} Q_l = 0$$

... nonconvex: use perspective formulation to "convexify"!

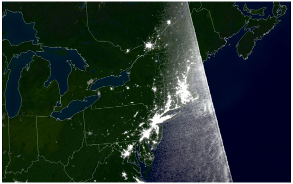
$$z_k \geq y_k \text{ and } y_k \geq \nu_k^2$$

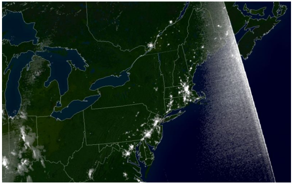


Integer & Nonlinear Optimization Applications

Mixed-Integer Nonlinear Program (MINLP)

$$\begin{array}{ll} \underset{x,y}{\text{minimize}} & f(x,y) \\ \text{subject to} & c(x,y) \le 0 \\ & y \in Y \text{ integer} \end{array}$$





Power Grid & Applications of MINLP:

- optimal power flow
- tertiary voltage control
- contingency analysis
- capacity expansion planning

Other Applications:

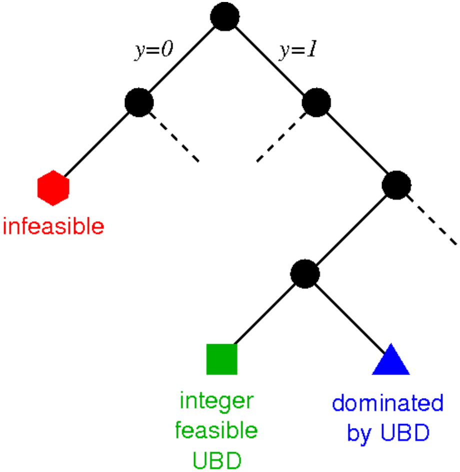
- reactor core reload operation
- gas transmission network design

AC Power Flow & Integer Nonlinear Optimization

Classical Branch-and-Bound [Dakin, 1965]

Solve relaxed NLP ($0 \leq y \leq 1$ continuous relaxation)
...solution value provides lower bound

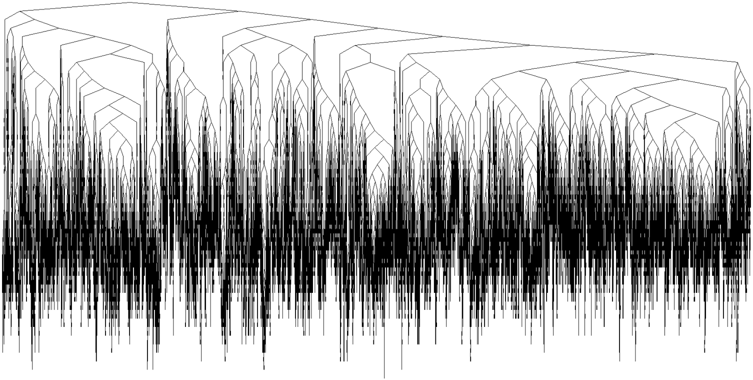
- Branch on y_i non-integral
- Solve NLPs & branch until
  1. Node infeasible: ●
  2. Node integer feasible: □
     $\Rightarrow$ get upper bound (U)
  3. Lower bound $\geq U$: ▲



Search until no unexplored nodes
Software:

- GAMS-SBB [GAMS] local
- MINLPBB [L] local
- BARON [Sahinidis] global
- Couenne [Belotti] global

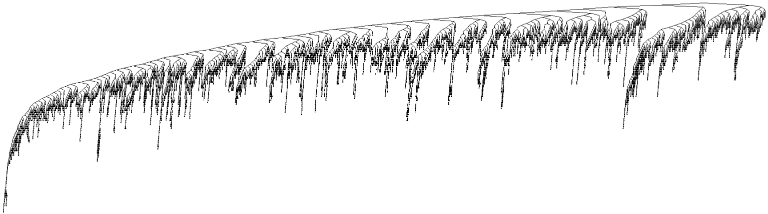
Challenges: MINLP Trees are Huge



MIP trees from `SCIP-1.2` [Achterberg et al.]

Exponential trees are generated on-the-fly $\Rightarrow$ little parallelism

Challenges: MINLP Trees are Huge

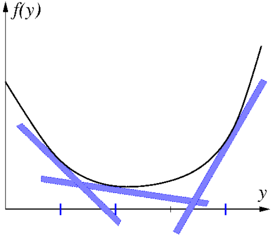


MIP trees from `SCIP-1.2` [Achterberg et al.]

Exponential trees are generated on-the-fly $\Rightarrow$ little parallelism

Other MINLP Software

Tremendous growth of new solvers in past 10 years based on outer approximation ideas.
... great for convex problems,
... challenging for nonconvex problems!



1. Outer Approximation
   - iterate between MILP and NLP solver
   - Solvers: `DICOPT++` [Grossmann], `BONMIN` [IBM/CMU]
2. LP/NLP-Based Branch-and-Bound
   - avoid re-solution of MILP tree-search
   - Solvers: `BONMIN` (open-source), `FilMINT` [L & Linderoth]
3. Extended Cutting Plane Method
   - avoid nonlinear solves (Kelley's cutting plane method)
   - Solver: α-`ECP` [Westerlund]

AC Power Flow & Integer Nonlinear Optimization

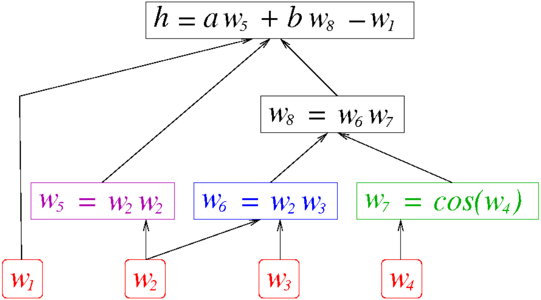
Decomposition of Nonlinear Functions

Consider AC-like function:

$$h(x_1, x_2, x_3, x_4) = ax_2^2 + bx_2x_3\cos(x_4) - x_1$$

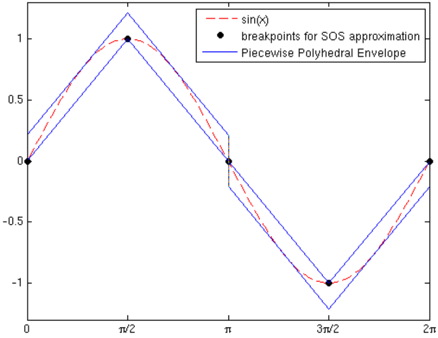
where a and b constants.

- $w_j = x_j \quad j = 1, \ldots,$
- $w_5 = w_2^2$
- $w_6 = w_2 w_3$
- $w_7 = \cos(w_4)$
- $w_8 = w_6 w_7$
- $h = aw_5 + bw_8 - w_1$



... decompose nonconvex function into univariate/bivariate component

Outer Approximation by Piecewise Polyhedral Envelopes



Univariate $w_h = h(w)$ becomes

$$\sum_{k \in I} \lambda_k \left(h(w_k) - L_k\right) \leq w_h \leq \sum_{k \in I} \lambda_k \left(h(w_k) + U_k\right)$$

... introduces special-ordered set (SOS) variables λ_k

Piecewise Polyhedral Envelopes

Obtain bound L_k by solving

$$L_k = \max_{w \in [w^k, w^{k+1}], \lambda^k + \lambda^{k+1} = 1} \left(0, \lambda^k h(w^k) + \lambda^{k+1} h(w^{k+1}) - h(w) \right)$$

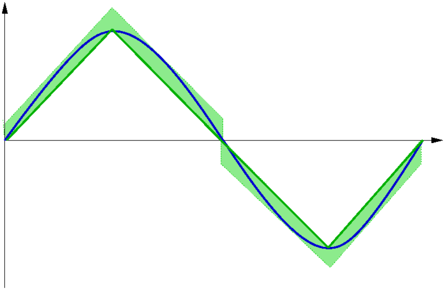
... similar for U_k

Bounds L_k, U_k pre-computed on $[w_k, w_{k+1}]$, e.g. $h(w) = w^2$:

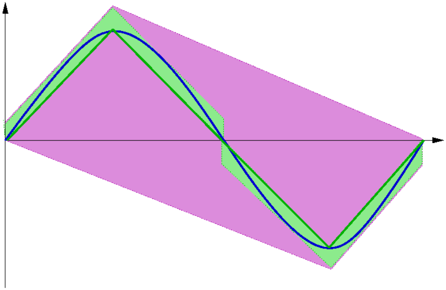
$$L_k = (w_{k+1} - w_k)^2/4, \qquad U_k = 0$$

... easy to get bounds for other functions.

Piecewise Envelope Problem: Illustration



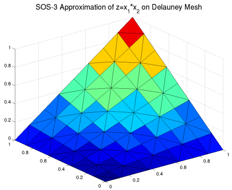
SOS Outer Approximation



Convex Hull

Branch-and-Refine: Outline

Ideas generalize to 2D functions



Classical Branch-and-Bound:
Solve envelope problem (E) branch on SOS-condition or $y \in \mathbb{Z}^t$

$\Rightarrow$ large discretization error or large number of λ_k variables

Idea: Instead refine discretization after branching:

- tighten envelope as we go down tree
- exploit exactness of bilinear terms $w_1 w_2$
- better numerical results

Branch-and-Refine: Branching

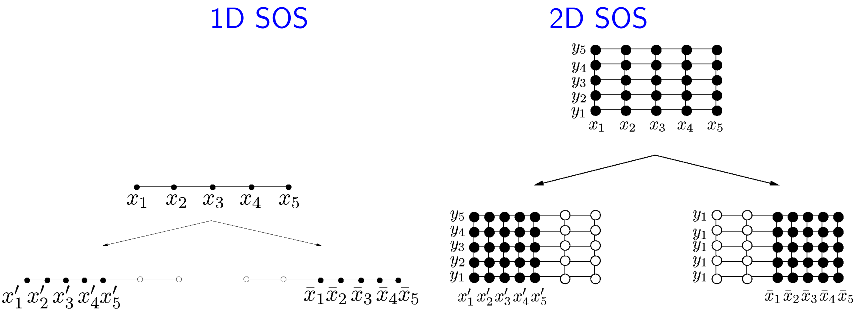


Illustration of branching and refinement

Test Problems (Tertiary Voltage Control)

Data and models from Tractebel Engineering (Belgium).

prob	#var	#cons	#var OA	#cons OA	#sets λ	#disc
TVC1	16	9	269	200	39	6
TVC2	18	9	275	204	40	6
TVC3	27	15	422	315	61	9
TVC4	27	15	422	315	61	9
TVC5	37	21	602	449	87	13
TVC6	38	21	635	472	92	14

- moderately sized problems (4, 7, 10 buses)
- BARON cannot solve any instance
- `MINLPBB` & `BONMIN-BB` give global minimum

Implementation Details & Tricks

- LPs solved by CPLEX, NLPs solved by filterSQP (old)
- decomposition hand-coded ... can be automated
  - exploit common sub-expressions ... reduce SOS-variables
  - can be automated, similar to automatic differentiation (AD)
- adaptive bound tightening & propagation ... at every node
  - propagate & strengthen bounds through computational graph
  - pre-solve (LP) to reduce range of variables
    $\min / \max w_{ij}$ subject to outer approximation
    ... tightens range of w_{ij}
- adapt branch-and-bound rules: include approximation error
  - (generalized) pseudo-cost branching
  - (generalized) best-estimate node selection

Argonne is developing MINOTAUR for nonconvex MINLPs

Preliminary Numerical Results (# LPs solved)

prob	basic	+presolve	+var-select	+node-select
TVC1	108861	40446	7756	8031
TVC2	fail	72270	5792	5547
TVC3	62045	861	627	627
TVC4	fail	38792	1396	1582
TVC5	fail	7369	5619	4338
TVC6	fail	12131	6096	5503

- exploiting structure and clever branching techniques
- can solve nonconvex MINLPs to proven global optimality
- only way I know to “prove” there is no solution

Conclusions & Summary

Mixed-Integer Nonlinear Optimization Solvers

1. Many new solvers in past 10 years: `BONMIN`, `FilMINT`, `SBB`, ...
2. Local optimization of AC power flow equations is standard.
   Most nonlinear optimizers are excellent solvers too!
3. (`BONMIN-BB` and `MINLPBB`) work for AC-OPF
   ... no global guarantees
4. Global optimization of AC power flow remains challenging.
5. Try MINLP solvers on NEOS, see `http://neos.mcs.anl.gov/`

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MINOTAUR

It's only half bull

Argonne's new nonconvex solver: MINOTAUR.

- Exploit structure: discrete & nonlinear.
- Efficient and reliable nonlinear optimizers.
- `C++` implementation.